LESSON 3.7

Using Systems of Inequalities to Find the Feasible Region

CAREER SPOTLIGHT: Computer Network Support Specialists

Occupation Description

Computer network support specialists analyze, troubleshoot, and evaluate computer network problems. They play an important role in the routine maintenance of their organization's networks, such as performing file backups on the network. Maintenance can be performed daily, weekly, or monthly and is important to an organization's disaster recovery efforts. Network support specialists may assist computer users through phone, email, or in-person visits. They often work under network and computer systems administrators, who handle more complex tasks.

Education

For computer network support specialists, many employers accept applicants with an associate's degree, although some prefer applicants to have a bachelor's degree.

Positions that are more technical are likely to require a degree in a field such as computer science, engineering, or information science, but for others, the applicant's field of study is less important.

Potential Employers

The largest employers of computer network support specialists are as follows:

Computer systems design and related services	19%
Telecommunications	11%
Finance and insurance	7%
Management of companies and enterprises	6%
Data processing, hosting, and related services	4%

Watch a video about computer network support specialists: https://cdn.careeronestop.org/OccVids/OccupationVideos/15-1151.00.mp4

Career Cluster
Information Technology

Career Pathway
Network Systems

Career Outlook

- Salary Projections:
  Low-End Salary, $40,620
  Median Salary, $65,450
  High-End Salary, $110,450
- Jobs in 2019: 195,100
- Job Projections for 2028: 207,700 (Increase of 6%)

Algebra II Concepts

- Write and solve systems of inequalities from real-world situations.
- Demonstrate how computer network specialists might apply systems of inequalities.

Is this a good career for me?
Computer network specialists typically do the following:

- Create electronic data backup to prevent loss of information.
- Implement security measures for computer or information systems.
- Resolve computer network problems.
- Configure computer networks.
- Install computer software.

Lesson Objective

In this lesson, you will learn about using systems of linear inequalities to find the feasible region for the solutions to problems.

- You will write and solve systems of inequalities from real-world situations.
- You will demonstrate how computer network specialists might apply systems of inequalities.
- You will find the feasible region where solutions to problems may be found.

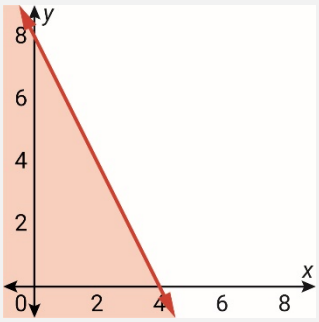
Systems of Linear Inequalities

A **system of linear inequalities** is a system of two or more linear inequalities in two variables. To solve a system of linear inequalities, graph each of the linear inequalities on the Cartesian Plane. The overlapping solution regions of the inequalities define the **feasible region**, or set of all ordered pairs for which all of the inequalities in the system are true.

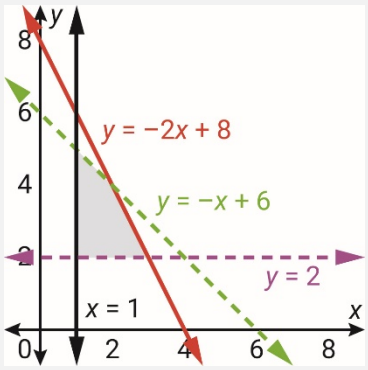
For example, consider the system of linear inequalities:

$$y \leq -2x + 8$$
$$y < -x + 6$$
$$x \geq 1$$
$$y > 2$$

To graph an inequality in two variables, begin by graphing its corresponding equation. Use a solid line for the ≤ or ≥ cases and a dashed line for the < or > cases. After graphing the line, shade the **half-plane** that corresponds to the solution region for that inequality. For the inequality $y \leq -2x + 8$, graph the solid line $y = -2x + 8$, then shade the half-plane below it.



Repeating this process for the other three inequalities results in the following graph. Only the feasible region is shaded to make the graph clearer.



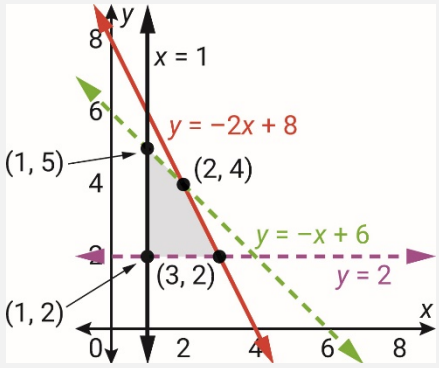
Many times, the boundary points of the feasible region are used to solve application problems. In order to find the coordinates of boundary points, set the equations equal to each other and solve.

$$\begin{aligned} -2x + 8 &= -x + 6 \\ 8 &= x + 6 \\ x &= 2 \end{aligned}$$

Substituting $x = 2$ into either equation results in the ordered pair (2, 4). For vertical or horizontal lines, substitute the known value into the second equation and solve. For instance, the boundary point of the lines $y = -2x + 8$ and $y = 2$ is:

$$\begin{aligned} -2x + 8 &= 2 \\ -2x &= -6 \\ x &= 3 \end{aligned}$$

This results in the boundary point at (3, 2). The other boundary points are labeled on the graph.



1 Step Into the Career: Applying Systems of Two Linear Inequalities

Regina is monitoring the bandwidth of a company's local computer network as part of her job as a computer network support specialist. Her company has tablets, x, and laptop computers, y, and a bandwidth maximum of 100 Megabits per second (Mbps).

During the day, the tablets require 4 Mbps and the laptops require 10 Mbps. At night, when software updates occur, the tablets require 8 Mbps and the laptops require 5 Mbps. In addition, the company has a maximum of 10 tablets and 8 laptops available.

Determine the system of linear inequalities that models this situation. Sketch this system of inequalities and sketch the feasible region of possibilities for the combinations of tablets and laptops that may be used at any one time.

Devise a Plan

Step 1: Define the system of linear inequalities that models the problem.

Step 2: Sketch the graph of the system of linear inequalities.

Step 3: Shade the feasible region of solution points for the system of inequalities.

Walk Through the Solution

Step 1: Given the definition of the x and y variables, the inequality describing the amount of bandwidth used during the day is:

$$4x + 10y \leq 100$$

The inequality describing the amount of bandwidth used during the night is:

$$8x + 5y \leq 100$$

The inequalities describing the number of tablets and laptops available are:

$$0 \leq x \leq 10$$

$$0 \leq y \leq 8$$

Notice, the variables each have a minimum value of 0 because a negative number of devices is not possible.

This completes the system of inequalities:

$$4x + 10y \leq 100$$

$$8x + 5y \leq 100$$

$$0 \leq x \leq 10$$

$$0 \leq y \leq 8$$

Step 2: Sketch the graphs of the inequalities. The first two inequalities are written in general form. So, you can use the x and y axes' intercepts. For example, when graphing the linear equation $4x + 10y = 100$, the intercepts may be found by setting each variable equal to 0.

y-intercept: $10y = 100$, therefore $y = 10$ and the y-intercept is (0, 10).

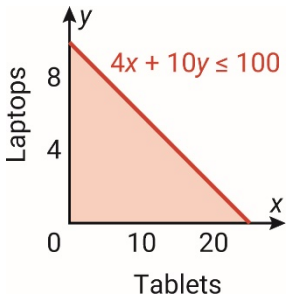
x-intercept: $4x = 100$, therefore $x = 25$ and the x-intercept is (25, 0).

The other option is to write each inequality in slope-intercept form by solving for y.

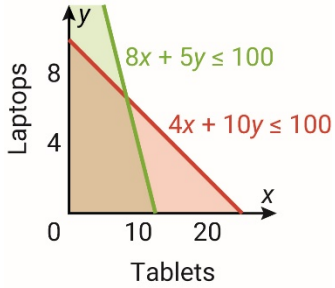
$$4x + 10y \leq 100$$

$$10y \leq -4x + 100$$

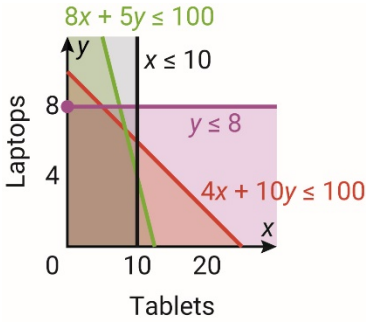
$$y \leq -0.4x + 10$$



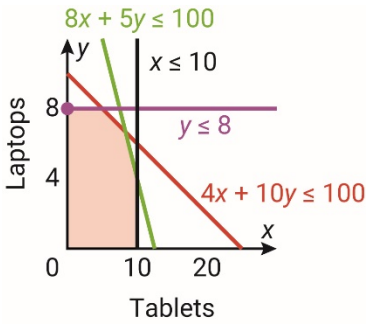
Next, graph the inequality $8x + 5y \leq 100$, which is equivalent to $y \leq -1.6x + 20$.



Sketch the graphs and shade the solution regions for the inequalities $0 \leq x \leq 8$ and $0 \leq y \leq 8$ as well. Only the regions in Quadrant I are shaded here because there will not be a negative number of tablets or laptops.



Step 3: Highlight the feasible region of the graph. This is the overlapping shaded region of the individual inequalities solution sets. Only the feasible region is shaded below.



Finally, the boundary points of the feasible region may be found by finding the appropriate intersection points of the lines. To find the intersection between the first two equations, solve the system of linear equations using the elimination method.

$$4x + 10y = 100$$

$$8x + 5y = 100$$

Multiply the first equation by 2. Then subtract the equations to eliminate the x-variable.

$$8x + 20y = 200$$

$$\underline{-\ (8x + 5y = 100)}$$

$$15y = 100$$

Solve for y, then back-substitute to find the value of x.

$$y = \frac{100}{15} = \frac{20}{3} \approx 6.67$$

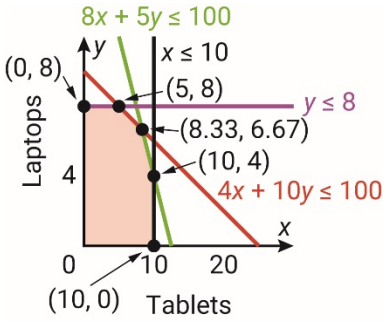
$$4x + 10\left(\frac{20}{3}\right) = 100$$

$$4x + \frac{200}{3} = \frac{300}{3}$$

$$4x = \frac{100}{3}$$

$$x = \frac{25}{3} \approx 8.33$$

The remainder of the boundary points are found by substituting the known values into the second equation. This results in five boundary points on the feasible region.



On the Job: Applying Systems of Two Linear Inequalities

1. Shasta, in her position as a computer network support specialist, is analyzing the bandwidth limitations for a client company. The client has tablets, x, and laptops, y, that use the internet bandwidth in the office.

   During the daytime, the company has a total of 71 Megabytes per second (Mbps) of available bandwidth. The tablets use 3 Mbps and the laptops 8 Mbps during this time.

   During the evening, the company leases additional bandwidth, to give them a total of 87 Mbps available. During this time, the tablets use 9 Mbps and the laptops use 6 Mbps as they upload information and receive updates.

   Write the system of linear inequalities that represents this problem. Sketch the system of inequalities and shade the feasible region. Label all boundary points of the feasible region. Remember, there cannot be a negative number of tablets or laptops.

2 Step Into the Career: Applying Systems of Two Linear Inequalities

Victor is helping configure the email server as part of his job as a computer network support specialist. A regular email has a maximum size of 30 megabytes (Mb). For this type of email, pictures have an average size of 4 Mb and data tables have an average size of 2 Mb. A multimedia email has a maximum size of 50 Mb. For this type of email, pictures have an average size of 5 Mb and data tables have an average size of 5 Mb.

Let *x* represent the number of pictures and *y* the number of data tables in each email. State the system of linear equations representing this problem. Graph the system, shade the feasible region, and identify the boundary points of the feasible region. Remember, the number of pictures and data tables in an email cannot be a negative number.

Devise a Plan

Step 1: Define the system of linear inequalities that models this problem.

Step 2: Graph the system of inequalities, shading the feasible region.

Step 3: Determine the boundary points of the feasible region.

Walk Through the Solution

Step 1: Given the definition of the x and y variables, the inequality for a regular email size is:

$$4x + 2y \leq 30$$

The inequality for a multimedia email is:

$$5x + 5y \leq 50$$

Since the number of pictures and data tables cannot be negative, the system of linear inequalities is:

$$4x + 2y \leq 30$$

$$5x + 5y \leq 50$$

$$x \geq 0$$

$$y \geq 0$$

In order to graph the system, the first two inequalities are written in slope-intercept form.

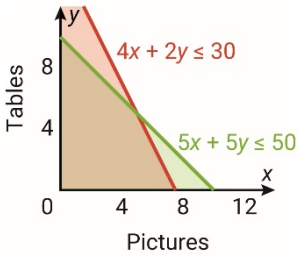
$$y \leq -2x + 15$$

$$y \leq -x + 10$$

$$x \geq 0$$

$$y \geq 0$$

Step 2: Graph the linear inequalities in quadrant I of the Cartesian plane.



Step 3: In order to find the boundary point of the first two inequalities, solve the resulting linear system. When the equations of the boundary lines are in slope-intercept form, the system may be solved by substitution.

$$y = -2x + 15$$

$$y = -x + 10$$

$$-2x + 15 = -x + 10$$

$$15 = x + 10$$

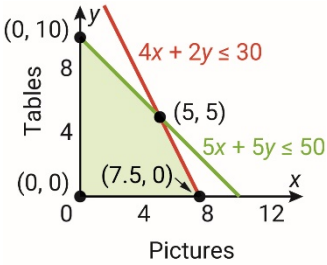
$$x = 5$$

Substitute $x = 5$ into one of the first two equations for the boundary lines:

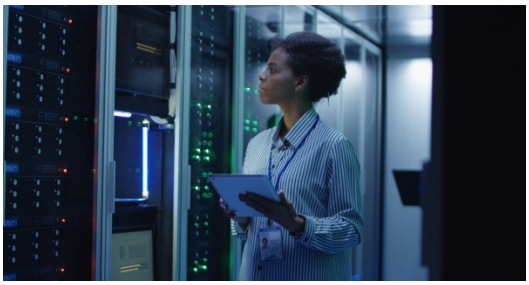
$$y = -5 + 10 = 5$$

The boundary point is (5, 5).

The remainder of the boundary points are found as *x* or *y* intercepts of the boundary lines. This results in the points (0, 10) and (7.5, 0).



On the Job: Applying Systems of Two Linear Inequalities



2. Joshua is managing the message limits for his company's email server as part of his job as a computer network support specialist. The messages being sent include audio attachments, x, and video attachments, y.

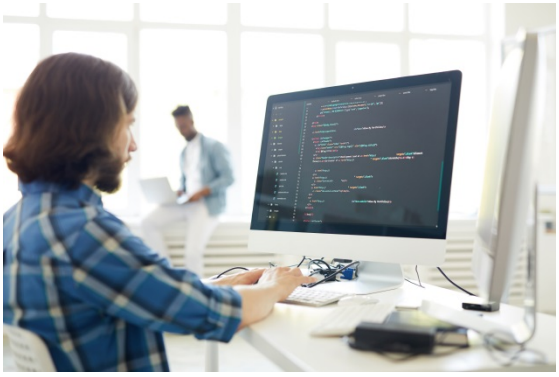
A regular email message has a size limit of 60 megabytes (Mb). An average audio attachment for a regular message is 7 Mb and an average video attachment is 6 Mb.

A zipped email message has a size limit of 27 Mb. An average zipped audio attachment is 4 Mb and an average zipped video attachment is 1 Mb.

Define the system of linear inequalities describing this problem. Sketch the graph of the system. Shade the feasible region and identify its boundary points.

Career Spotlight: Practice

3. Christian is analyzing the bandwidth usage of his network as part of his job as a computer network support specialist. He finds laptop computers use 6 megabytes per second (Mbps) and desktop computers use 4 Mbps. The bandwidth for the company is 120 Mbps.



State the linear inequality that represents this problem, given *x* is the number of laptops and *y* is the number of desktops. Sketch the graph of the system of inequalities on the plane, knowing the number of each type of computer cannot be negative.

4. Haadiya is managing the email server at her company as a computer network support specialist. The emails have a maximum size limit of 48 megabytes (Mb). Photos have an average size of 3 Mb and videos have an average size of 4 Mb.

Given x is the number of photos and y is the number of videos, state the inequality representing this problem. Given that the number of photos and videos must be at least 0, sketch the graph of the system of inequalities.

5. Tamara, in her job as a computer network support specialist, is purchasing software for her company. Software x has a cost of $8 per copy, software y has a cost of $12 per copy, and Tamara has $120 in the budget to purchase software this month. In addition, she needs at least 3 copies of software *x* and no more than 6 copies of software *y*.

State the system of linear inequalities modeling this problem. Sketch the system, shading the feasible region and labeling its boundary points.

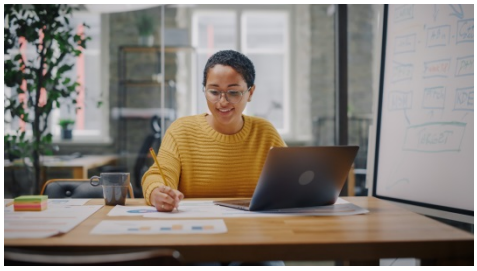
Devise a Plan

Step 1: Define the system of linear inequalities.

Step 2: ____?____

Step 3: ____?____

Step 4: ____?____



Career Spotlight: Check

6. Dara is trying to determine options for bandwidth usage at her office as its computer network support specialist. Provide her four points that are in the solution set to the following system of inequalities:

$$3x + 6y \leq 120$$

$$x > 5$$

$$y \geq 0$$

Point	Coordinates
P	
Q	
R	
S	

7. Nam is managing the cloud storage at his company as its computer network support technician. He determines that given *x* video and *y* audio files, the following system of inequalities reflects his company's current situation.

$$5x + 3y \leq 30$$

$$4x + 2y \leq 22$$

$$x \geq 0$$

$$y \geq 0$$

Select all the ordered pairs that are in the solution set to this system of inequalities.

a. (–1, 5)

b. (5, 4)

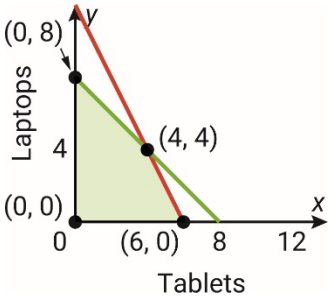
c. (3, 2)

d. (1, 7)

e. (0, 7)

f. (4, -1)

8. Sandra is analyzing the bandwidth usage of the tablets, x, and laptops, y, as part of her job as a computer network support technician. She is presented with the following graph that shows the possibilities given their current bandwidth.



State the system of equations represented by this graph.

9. Yaalini, in her position as a computer network support specialist, is analyzing the bandwidth limitations for a client company. The client has tablets, x, and laptops, y, that use the internet bandwidth in the office.

During the daytime, the company has a total of 84 Megabytes per second (Mbps) of available bandwidth. The tablets use 4 Mbps and the laptops 7 Mbps during this time.

During the evening, the company leases additional bandwidth, to give them a total of 100 Mbps. During this time, the tablets use 10 Mbps and the laptops use 5 Mbps as they upload information and receive updates.

Given that the number of tablets and laptops cannot be negative, select all the inequalities that make this system.

a. $y > 0$

b. $x \geq 0$

c. $10x + 5y \leq 100$

d. $4x + 7y \leq 84$

e. $7x + 4y \leq 100$

f. $5x + 10y \leq 100$

g. $y \geq 0$

10. Samuel, in his job as a computer network support specialist, is purchasing software for his company. Software *x* has a cost of $12 per copy, software *y* has a cost of $10 per copy, and Samuel has $240 in the budget to purchase software this month. In addition, he needs at least 3 copies of software *x* and no more than 18 copies of software *y*.

Select all the following that are boundary points of the feasible region of this system of linear inequalities.

a. (0, 24)

b. (20, 0)

c. (3, 18)

d. (0, 18)

e. (10, 12)

f. (5, 18)

g. (3, 0)