LESSON 9.4

Applying Graphs of Quadratic Functions

CAREER SPOTLIGHT: Aerospace Engineer

Occupation Description

Aerospace engineers design primarily aircraft, spacecraft, satellites, and missiles. In addition, they create and test prototypes. They may develop new technologies for use in aviation, defense systems, and spacecraft.

Aerospace engineers often become experts in one or more related fields: aerodynamics, thermodynamics, materials, celestial mechanics, flight mechanics, propulsion, acoustics, and guidance and control systems. They typically specialize in one of two types of engineering: aeronautical or astronautical.

Education

Entry-level aerospace engineers usually need a bachelor's degree. Bachelor's-degree programs include classroom, laboratory, and field studies in subjects such as general engineering principles, propulsion, stability and control, structures, mechanics, and aerodynamics, which is the study of how air interacts with moving objects.

Potential Employers

The largest employers of aerospace engineers are as follows:

Aerospace product and parts manufacturing	35%
Federal government, excluding postal service	15%
Engineering services	15%
Navigational, measuring, electromedical, and control instruments manufacturing	10%
Research and development in the physical, engineering, and life sciences	9%

Watch a video about aerospace engineers: https://cdn.careeronestop.org/OccVids/OccupationVideos/17-2011.00.mp4

Career Cluster

Science, Technology, Engineering & Mathematics

Career Pathway

Engineering and Technology

Career Outlook

- Salary Projections:
  Low-End Salary, $72,450
  Median Salary, $116,500
  High-End Salary, $166,620
- Jobs in 2018: 67,200
- Job Projections for 2028: 68,300 (increase of 2%)

Algebra Concepts

- Graph quadratic functions.
- Use factoring and completing the square to identify characteristics of parabolas.

Is this a good career for me?

Aerospace engineers:

- Create models of engineering designs or methods.
- Test performance of electrical, electronic, mechanical, or integrated systems or equipment.
- Design electromechanical equipment or systems.
- Evaluate designs or specifications to ensure quality.

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Lesson Objective

In this lesson, you will look at how an aerospace engineer uses quadratic functions to model the path of a flight or the forces acting on an aircraft.

Graphing a Quadratic Function

The graph of a quadratic function is a **parabola**. It has a **line of symmetry** that passes through the **vertex**. A parabola can have 0, 1, or 2 ***x*-intercepts.**

For a parabola that opens down, the *y*-coordinate of the vertex is the **maximum** value of the function.

For a parabola that opens up, the *y*-coordinate of the vertex is the **minimum** value of the function.

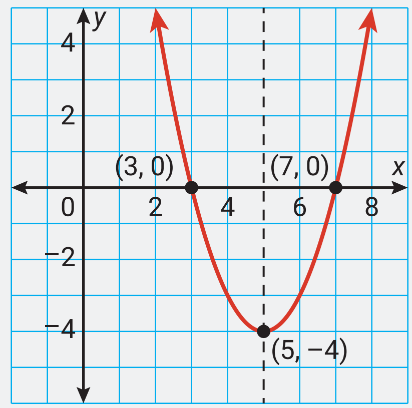
For the function shown in the graph, the vertex is $(5, -4)$, and -4 is the minimum.

The **vertex form** of a quadratic function with vertex (h, k) is $f(x) = a(x - h)^2 + k$.

The vertex form of the function shown is $f(x) = (x - 5)^2 - 4$.

The **intercept form** of a quadratic function with *x*-intercepts p and q is $f(x) = a(x - p)(x - q)$.

The intercept form of the function shown is $f(x) = (x - 3)(x - 7)$. The *x*-intercepts are 3 and 7.

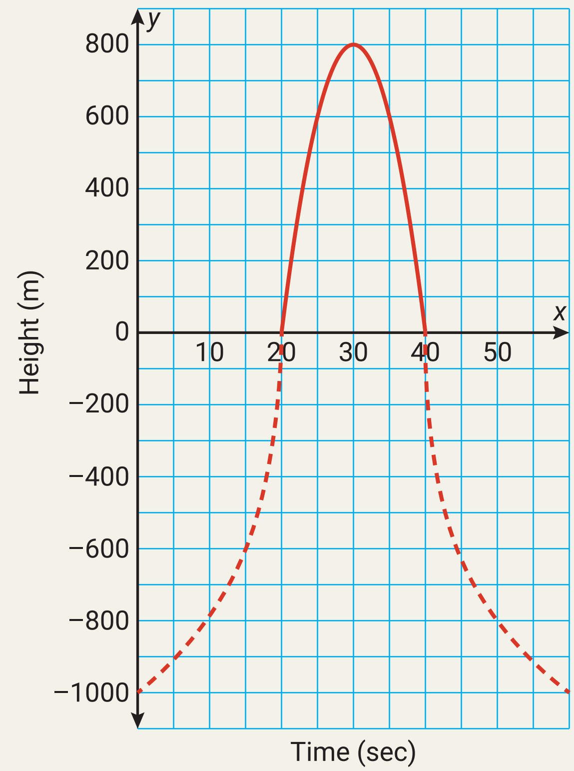


$f(x) = x^2 - 10x + 21$

1 Step Into the Career: Characteristics of a Parabola

When a spacecraft travels beyond Earth's atmosphere, the gravitational force is close to zero. To simulate microgravity on Earth, aerospace engineers use a special type of aircraft. The pilot flies the aircraft upward and then cuts the thrust, allowing the aircraft to continue rising and then fall back down in a parabolic pattern before increasing the thrust again.

The graph shows the height in meters of the aircraft as a function of time in seconds after the pilot begins to fly upward. The height is measured relative from when the pilot cuts the thrust. The solid line indicates the part of the flight that is parabolic, when the gravity is near zero. Identify the vertex and *x*-intercepts of the parabola, and interpret what each represents in this situation. Then write a quadratic function in intercept form for the parabola.



Devise a Plan

Step 1: Identify and interpret the vertex.

Step 2: Identify and interpret the *x*-intercepts of the parabola.

Step 3: Write a function for the parabola.

Walk Through the Solution

Step 1: Identify and interpret the vertex.

The vertex is (30, 800). It represents the maximum height of the jet, 800 meters above the point where the pilot cut the thrust. The jet reaches that point 30 seconds after the beginning of the maneuver.

Step 2: Identify and interpret the *x*-intercepts of the parabola.

The *x*-intercepts are 20 and 40. They represent the beginning and ending times of the parabolic portion of the flight. The pilot cuts the thrust 20 seconds into the maneuver and increases the thrust again at 40 seconds.

Step 3: Write a function for the parabola.

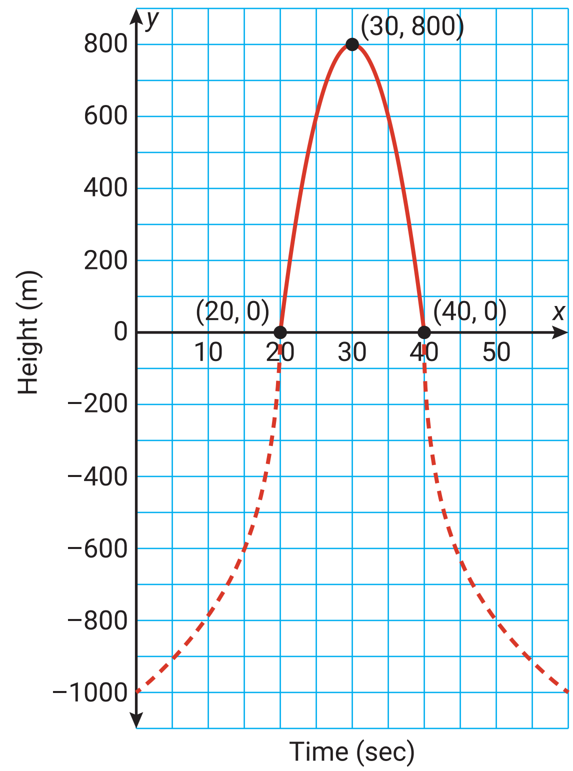
To write a function for the parabola in intercept form, substitute 20 and 40 for p and q in $f(x) = (x - p)(x - q)$.

$$f(x) = a(x - p)(x - q)$$
$$= a(x - 20)(x - 40)$$

Then use the vertex, (30, 800), to solve for a.

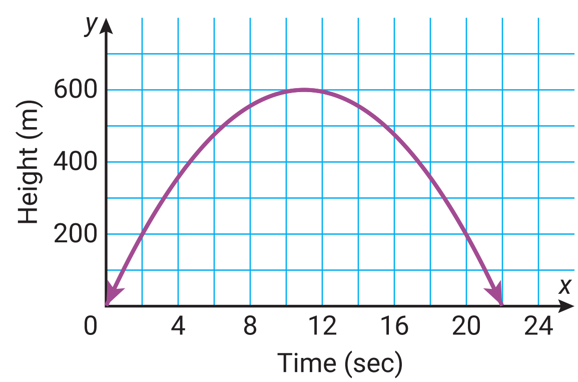
$$800 = a(30 - 20)(30 - 40)$$
$$800 = a(10)(-10)$$
$$800 = -100a$$
$$-8 = a$$

The function in intercept form for the parabola is $f(x) = -8(x - 20)(x - 40)$.



On the Job: Apply Characteristics of a Parabola

1. An aerospace engineer graphs the height in meters of a jet during a parabolic maneuver in relation to the time in seconds. Both the height and time are measured from the moment when the pilot cuts the thrust.



a. Does the vertex of the parabola represent a maximum or a minimum? Give the coordinates of the vertex, and explain what information they give in this situation.

b. What are the x-intercepts? What does each x-intercept represent?

c. Write the function in intercept form.

2 Step Into the Career: Quadratic Functions in Intercept Form

An aerospace engineer represents the height in meters of an aircraft during a parabolic maneuver, relative to the height when the pilot reduces the thrust, using the function $f(x) = -8(x - 20)(x - 40)$, where x is the time in seconds. Suppose the pilot cuts the thrust when the aircraft is at an altitude of 9000 meters. The engineer writes a new function where y represents the altitude of the aircraft. What is the function in intercept form for the altitude of the jet that the engineer writes? Then graph the new function.

Devise a Plan

Step 1: Write a function for the altitude of the aircraft.

Step 2: Rewrite the function in intercept form.

Step 3: Graph the function.

Walk Through the Solution

Step 1: Write a function for the jet's altitude.

For any height given by the original function, the altitude of the aircraft is 9000 meters greater. So, the altitude is given by the function $f(x) = -8(x - 20)(x - 40) + 9000$.

Step 2: To rewrite the function in intercept form, first simplify and then factor.

$$\begin{aligned} f(x) &= -8(x - 20)(x - 40) + 9000 \\ &= -8x^2 + 480x - 6400 + 9000 \\ &= -8x^2 + 480x + 2600 \\ &= -8(x^2 - 60x - 325) \\ &= -8(x + 5)(x - 65) \end{aligned}$$

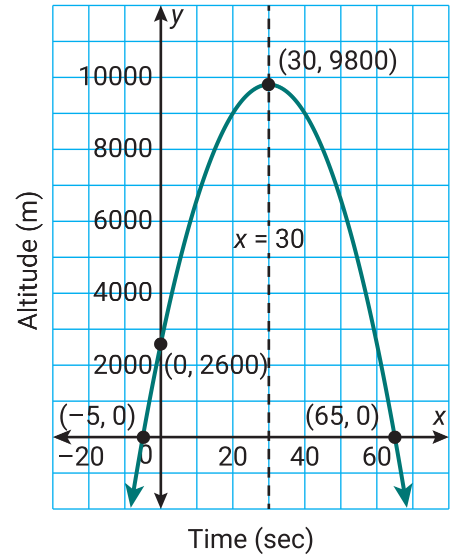
Photo credit: Sergey Kohl/Shutterstock.com

Step 3: Graph the function.

To graph the function, first plot the intercepts at (–5, 0) and (65, 0). Then locate the line of symmetry halfway between the intercepts, $x = 30$. Substitute 30 for x into the function to find the y-coordinate of the vertex.

$$\begin{aligned} f(x) &= -8(x + 5)(x - 65) \\ &= -8(30 + 5)(30 - 65) \\ &= -8(35)(-35) \\ &= 9800 \end{aligned}$$

Plot the vertex at (30, 9800). Use the vertex, intercepts, and line of symmetry to draw the parabola.



On the Job: Apply Quadratic Functions in Intercept Form

2. An aerospace engineer writes a function for the altitude in meters of a plane during a parabolic maneuver. The function is $f(x) = -6(x - 10)^2 + 9600$. The maneuver begins when $x = 0$ and lasts for 20 seconds.

a. What is the vertex of the parabola? Explain what information it gives in this situation.

b. Write the function in intercept form. What are the x-intercepts?

c. Graph the function. Highlight the part of the graph that shows the altitude of the plane.

3 Step Into the Career: Quadratic Functions in Vertex Form

In order to design aircraft, aerospace engineers need to understand the forces that act on an object as it moves through the air. *Thrust* is the force that moves an airplane forward. *Drag* is the force that works against thrust, slowing the airplane down. An aerospace engineer writes the function $f(x) = 0.15x^2 - 27x + 1500$ for the drag y, in pounds, of an aircraft as a function of airspeed x, in knots. At which airspeed is there the least amount of drag?

Photo credit: muratart/Shutterstock.com; muratart/Shutterstock.com

Devise a Plan

Use a graph of the function to find the minimum amount of drag.

Step 1: Rewrite the function in vertex form.

Step 2: Graph the function.

Step 3: Find the airspeed that produces the minimum amount of drag.

Walk Through the Solution

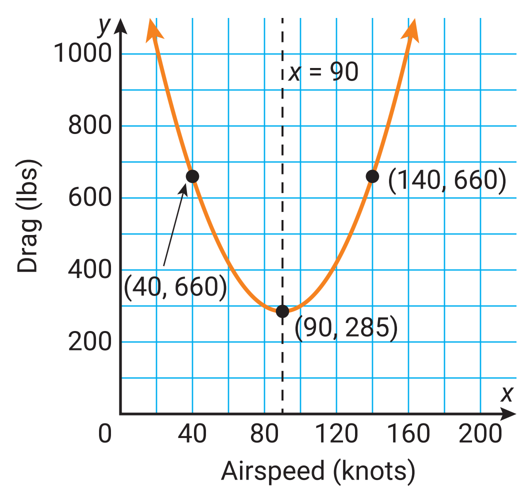
Step 1: Rewrite the function in vertex form by completing the square.

$$\begin{aligned} f(x) &= 0.15x^2 - 27x + 1500 \\ &= 0.15(x^2 - 180x) + 1500 \\ &= 0.15(x^2 - 180x + 8100) + 1500 - 0.15(8100) \\ &= 0.15(x - 90)^2 + 285 \end{aligned}$$

Step 2: Graph the function.

To graph the function, first plot the vertex at (90, 285). Then substitute a value for x to find another point on the graph.

$$\begin{aligned} f(x) &= 0.15(x - 90)^2 + 285 \\ &= 0.15(40 - 90)^2 + 285 \\ &= 0.15(-50)^2 + 285 \\ &= 0.15(2500) + 285 \\ &= 660 \end{aligned}$$



The graph passes through the point (40, 660). Draw the left half of the parabola through (40, 660) and ending at the vertex. The line $x = 90$ is a line of symmetry, so draw the right half of the parabola to mirror the left.

Step 3: Find the airspeed that produces the minimum amount of drag.

The vertex represents the minimum value of the function. The least amount of drag on the plane is 285 pounds, when the airspeed is 90 knots.

On the Job: Apply Quadratic Functions in Vertex Form

3. An aeronautical engineer is calculating the different forces on an airplane. The function $f(x) = 0.08x^2 - 12x + 680$ gives the drag for the plane, in pounds, as a function of airspeed, in knots.

a. Rewrite the function in vertex form. What is the vertex of the parabola?

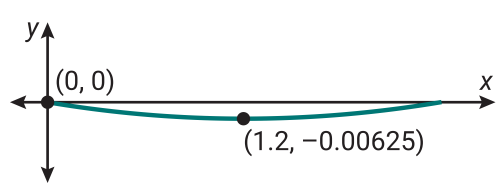
b. What is the *y*-intercept of the graph?

c. Graph the function. Label the line of symmetry and the points you used to draw the graph.

d. What is the significance of the vertex in this situation?

Career Spotlight: Practice

4. Astronautical engineers design and maintain different kinds of spacecraft. The Hubble Telescope has been in space for over 30 years and is still in use. The main mirror of the telescope is a parabola, though it looks almost flat. The graph represents a cross-section of the mirror. The coordinates are given in meters.



a. Give the coordinates of the vertex and the equation for the line of symmetry. What does the vertex represent?

b. What are the *x*-intercepts of the parabola? What do these points represent?

c. How wide is the mirror in meters? How deep is the mirror in millimeters?

d. Write the function in vertex form for the parabola.

5. Aerospace engineers designed the solid rocket booster (SRB), which provided the thrust needed to launch the space shuttle. Two minutes after launch, the SRBs would be separated from the space shuttle and fall back to Earth. The function $f(x) = -15x^2 + 3x - 60$ approximates the distance in feet of one SRB below the space shuttle after separation, where x is the time in seconds after separation. Graph the function for $0 \le x \le 4$, and label the vertex of the parabola with its coordinates.

Devise a Plan

Step 1: Rewrite the function in vertex form.

Step 2: ___?___

Step 3: ___?___

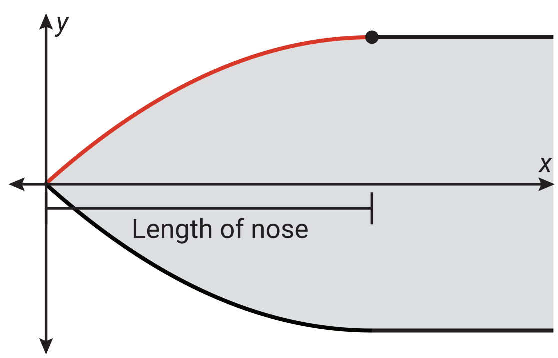
6. An aeronautical engineer uses a parabolic flight to simulate near-zero gravity. The altitude in meters of the plane during the parabolic maneuver is given by the function $f(x) = -8.3x^2 + 697.2x - 6142$, where x is the time in seconds. Graph the function. Label the line of symmetry with its equation, and label the vertex and x-intercepts with their coordinates.

QUICK TIP

First rewrite the function in intercept form by completing the square.

Career Spotlight: Check

7. When designing an airplane, an aeronautical engineer chooses a shape for the nose, the front part of the plane. Sometimes this shape is parabolic. For one airplane, the curve of the nose can be modeled by graphing the function $f(x) = -\frac{1}{81}x^2 + \frac{2}{3}x$, where both x and y are measured in feet. The vertex of the parabola is at the point where the nose meets the next part of the plane.



Select the answer from each box that makes the sentence true.

The vertex of the parabola is at ([**a.** 0 **b.** $\frac{1}{3}$ **c.** 27 **d.** 54] , [**a.** 0 **b.** $\frac{1}{9}$ **c.** 9 **d.** 36]). The nose of the plane is [**a.** 9 **b.** 27 **c.** 36 **d.** 54] feet long.

Photo credit: 3Dsculptor/Shutterstock.com

8. An aerospace engineer is using a parabolic flight to perform an experiment in microgravity. During the parabolic maneuver, the plane's height in meters with respect to the altitude at which the pilot cut the thrust is given by the function $f(x) = -8.1x^2 + 340.2x - 2592$, where x is the time in seconds after the pilot begins the maneuver.

Select all the statements that are true.

a. The vertex form of the function is $f(x) = -8.1(x - 21)^2 + 980.1$.

b. The x-intercepts represent the times when the plane is on the ground.

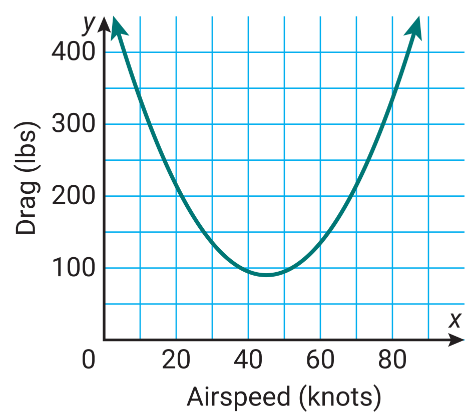
c. The x-intercepts are approximately 10 and 32.

d. The minimum height of the plane is 980.1 meters higher than where the parabolic maneuver began.

e. The graph of the function passes through the point (30, 324).

f. The graph of the function passes through (–21, 980.1).

9. An aeronautical engineer graphs the drag of an airplane as a function of its airspeed, as shown. What is a function for the graph?



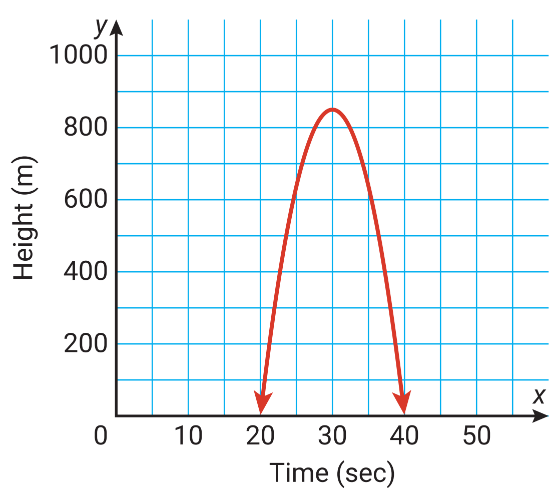
A. $f(x) = 0.2(x - 90)^2 + 45$

B. $f(x) = 0.2(x - 45)^2 + 90$

C. $f(x) = 0.2(x + 90)^2 + 45$

D. $f(x) = 0.2(x + 45)^2 + 90$

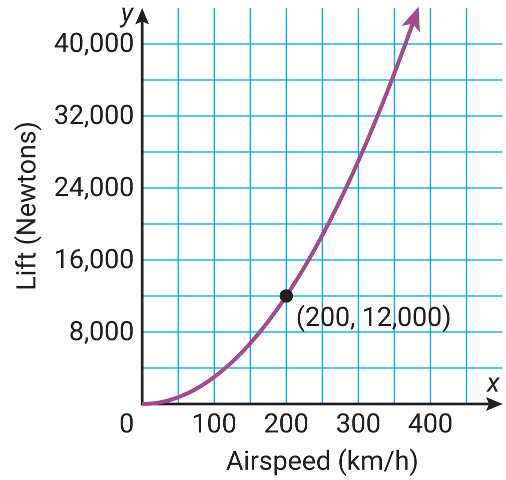
10. An experimental jet makes a high-altitude flight. An aerospace engineer models a portion of the flight using the graph shown. The height is the number of meters after the jet reaches an altitude of 9500 meters. The graph is in the shape of a parabola.



Determine whether each function represents the graph shown.

	Yes	No
$f(x) = -8.5(x - 20)(x - 40)$	☐	☐
$f(x) = -8.5(x - 30)^2 + 100$	☐	☐
$f(x) = -8.5(x + 20)(x + 40)$	☐	☐
$f(x) = -8.5(x - 30)^2 + 850$	☐	☐

11. *Lift* is the force that keeps an airplane in the air. An aeronautical engineer draws the graph at the right, which shows the lift of one plane as a function of its airspeed. The vertex of the parabola is at (0, 0). What is an equation for the function?



12. The function $f(x) = -6x^2 + 132x$ gives the height in feet of a plane throughout a parabolic maneuver relative to its altitude when the pilot cuts the thrust to begin the maneuver, where x is the number of seconds since the beginning of the maneuver. The aeronautic engineer who is using the flight to do research notes that the plane's altitude at the beginning of the maneuver is 8000 feet. What is a function for the plane's altitude during the maneuver?

Fill in the blanks by selecting values from the panel.

–11	726
–6	121
0	132
6	7274
11	8121
22	8132
66	8726

$f(x) = \_\_\_\_\_\_(x - \_\_\_\_\_\_)^2 + \_\_\_\_\_\_$

Notes